The Excess Enthalpy of (Steam + Ethane) in the Supercritical Region up to T = 699.4 K and p = 25.3 MPa

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Flow calorimetric measurements of the excess molar enthalpy $H_{\rm m}^{\rm E}$ of $(0.5 {\rm H_2O} + 0.5 {\rm C_2H_6})$ in the supercritical region carried out at temperatures from 573.7 to 699.4 K and at pressures in the range 5.05 to 25.3 MPa are reported. The measurements are fitted by a two-reference-fluid corresponding-states model which gives a good representation of the excess enthalpies up to the highest pressure.

KEY WORDS: excess enthalpy; fluid mixture; steam + ethane; supercritical region.

1. INTRODUCTION

This paper reports flow calorimetric measurements of the excess molar enthalpy H_m^E of (water + ethane)(g) made at pressures up to 25.3 MPa, temperatures up to 699.4 K, and the composition x = 0.5. These measurements parallel similar work [1] on (water + carbon dioxide)(g). Some measurements of H_m^E for (water + ethane) have already been reported. Lancaster and Wormald [2] used a low-pressure differential-flow mixing calorimeter to make excess enthalpy measurements at standard atmospheric pressure over the temperature range 363.2 to 393.4 K, and second virial cross coefficients B_{12} were derived from the results. At lower temperatures, Coan and King [3] measured the solubility of water in compressed ethane gas and obtained values of B_{12} down to 298 K. To fit these, it was necessary to use a temperature-

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dependent interaction parameter $(1 - k_{12}) = 0.974 + 0.09 \times 10^{18} T^{-8}$ (with T in K) in the pair potential combining rule $\varepsilon_{12} = (1 - k_{12})(\varepsilon_{11}\varepsilon_{22})^{1/2}$.

Lancaster and Wormald [4] then reported H_m^E measurements at temperatures from 448.2 to 698.2 K at pressures from 0.55 to 12.2 MPa. The values of H_m^E extrapolated back to standard atmospheric pressure were shown to be consistent with the measurements obtained using the lowpressure differential mixing calorimeter. Excess enthalpy isotherms were fitted to an empirical equation, and compression factors for the (0.5H₂O + 0.5C₂H₆)(g) mixture were calculated. The measurements at all pressures were well fitted using a cubic equation of state which was originally developed to fit properties of (water + alkane)(g) mixtures [5]. The phase diagram for (water + ethane) was determined by Danneil et al. [6] and is of Type IIIm in the classification of van Konynenburg and Scott [7]. The critical line originating at pure water ($T_c = 647.096$ K, $p_c = 22.064$ MPa) initially moves to lower temperatures and slowly rising pressures. The critical locus of the mixture passes through a minimum temperature at about 624 K; then the pressure increases rapidly with temperature.

2. EXPERIMENTAL

The high-pressure-flow mixing calorimetric apparatus was the same as that described previously [8] for measurements on (water + carbon dioxide). Liquid ethane at 253 K was confined over mercury and was pumped by displacing the mercury with water from a metering pump. The apparatus was back-pressured with nitrogen, and the pressure was controlled to ± 20 kPa. To obtain isothermal conditions in the mixing calorimeter, the heater was adjusted manually rather than by computer. Ethane of mole fraction purity (0.993C₂H₆) was used, and the water was ordinary degassed distilled water.

Our previous high-pressure measurements [4] were made at x = 0.5 over the pressure range 0.57 to 12.2 MPa and over the temperature range 448.5 to 698.2 K. The new measurements overlap this previous work and extend over the pressure range 5.15 to 24.95 MPa and the temperature range 573.7 to 699.4 K. All temperatures are on the IPTS-68 scale. The new measurements include a pressure scan at 650.7 K, just 3.6 K above the critical temperature of water. The measurements at this temperature, particularly at pressures in the range 21 to 24 MPa bracketing the critical pressure (22.064 MPa) of water, were very difficult to make, and under these experimental conditions great care had to be taken. There was no difficulty in adjusting the flow rates so as to obtain a composition of $x = (0.50 \pm 0.01)$, but in this region where H_m^E increases rapidly with pressure, very small fluctuations in temperature or pressure produced large perturbations of the steady state we were trying to obtain, and we often

had to monitor the apparatus for several hours until conditions stabilized sufficiently to allow reliable measurements. The measurements at the higher temperatures 674.2 and 699.4 K were easier to make. In all, we achieved a total of 46 measurements at pressures greater than 10 MPa, and these included 15 measurements at pressures greater than 20 MPa.

We estimate that the average uncertainty of all measurements up to 20 MPa is $\pm 4\%$. Beyond this pressure and for the measurements at 674.2 K, it is about the same; for those at 699.4 K, it is about $\pm 5\%$; and for those at 650.7 K and at the pressures 21.6 and 22.7 MPa, where the isotherm is steepest, it is about $\pm 6\%$. The random error in the measurements depends upon the amplitude of the temperature fluctuations monitored by the platinum resistance thermometers attached to the calorimeter. Where the isotherms shown in Fig. 1 are steep, low-pressure fluctuations, arising either in the flash boilers or in the pressure control system, produced larger than usual temperature fluctuations, and good measurements proved difficult to achieve. Our further concern was that there might be a systematic error in the measurements, particularly in the region where H_m^E was greater than $8 \text{ kJ} \cdot \text{mol}^{-1}$. As the mixing process is endothermic, we would expect heat to leak into the calorimeter rather than out of it, and if heat leaks are significant, they will result in low values



Fig. 1. The excess molar enthalpy $H_{\rm m}^{\rm E}$ of $(0.5H_2O + 0.5C_2H_6)(g)$ plotted as a function of the pressure *p*. (\bigcirc) This work (Table I); (-----) calculated from the two-fluid corresponding-states model.

of $H_{\rm m}^{\rm E}$. The only indication that systematic errors are in fact small comes from the analysis presented in the next section. The results are listed in Table I and are plotted in Fig. 1.

3. COMPARISON WITH THEORY

Our previous H_m^E measurements at pressures up to 12.2 MPa were all well fitted by a cubic equation of state [5] which we had developed to fit measurements on (water + alkane)(g) mixtures. This equation fits the new measurements at 699.4 K and up to 24.95 MPa well but fails to fit the measurements at 650.7 K at pressures higher than 21 MPa. As can be seen from Fig. 1, H_m^E in this region increases to almost $10 \text{ kJ} \cdot \text{mol}^{-1}$, but the equation from Ref. 5 (not shown in the figure) gave values of about $8 \text{ kJ} \cdot \text{mol}^{-1}$. Other cubic equations were tried, but with similar lack of success in this region. The inadequacy of analytic equations in the critical region is well known, and to obtain a description of a binary fluid mixture in this region, elaborate equations capable of describing the crossover from singular thermodynamic behavior at the critical point to regular behavior far away from the critical point have been proposed [9, 10]. However, to obtain the necessary parameters for these equations an extensive database of accurate measurements is needed, and even for a well-studied system such as (carbon dioxide + ethane) the inconsistencies between the different sets of measurements severely limit what can be achieved [10].

The lack of an adequate equation of state which is capable of fitting the thermodynamic surfaces of water and ethane and which has adequate combining rules for fitting the properties of the mixture left us with the concern that our measurements might be in error. Fenghour et al. [11] faced the same problem when they analysed their isochoric density measurements on (water + carbon dioxide). They tested the Peng–Robinson [12] equation of state and found it to be inadequate at high densities and for water-rich compositions. They then investigated the one-fluid and twofluid corresponding-states models suggested by Rowlinson and Watson [13, 14] and found the latter to be superior. A software package using the two-fluid corresponding-states model for the prediction of mixture properties has been developed, and as full details are given elsewhere [15, 16], only an outline is needed here.

In this model, the real fluid mixture consisting of components *i* and *j* is considered to be an ideal mixture of two hypothetical components. The residual Helmholtz energy $A_{\text{mix}}^{\text{res}}$ of the mixture is given by

$$A_{\rm mix}^{\rm res}(V,T,x) = \sum_{i} x_i A_i^{\rm res}(V,T)$$
(1)

where x_i is the mole fraction of component *i*, $A_i^{\text{res}}(V, T)$ is the residual Helmholtz energy of the *i*th hypothetical component, and V and T are the volume and temperature of the mixture, respectively. To utilize the principle of corresponding states, the residual Helmholtz energy of each hypothetical component is expressed in terms of the properties of a real reference fluid r_i so that

$$A_i^{\text{res}}(V, T) = f_i A_{r_i}^{\text{res}}(V/h_i, T/f_i)$$
⁽²⁾

where f_i and h_i are scaling parameters. For the van der Waals two fluid model the scaling parameters are given by

$$h_i = \sum_j x_j h_{ij} \tag{3}$$

$$f_i h_i = \sum_j x_j h_{ij} f_{ij} \tag{4}$$

The cross-term scaling parameters h_{ij} and f_{ij} are obtained using Lorentz–Berthelot combining rules.

$$h_{ij} = \eta_{ij} [0.5(h_{ii}^{1/3} + h_{jj}^{1/3})]^3$$
(5)

$$f_{ij} = \xi_{ij} (f_{ii} f_{jj})^{1/2} \tag{6}$$

The binary interaction parameters η_{ij} and ζ_{ij} are adjustable. All that is needed are equations of state which describe the Helmholtz energy of the two real fluids. For water, the equation of state of Saul and Wagner [17] was used, and for ethane the equation used was that of Friend et al. [18].

The density or enthalpy of the fluid mixture is obtained by taking the appropriate derivatives of the Helmholtz energy, and the adjustable parameters η and ξ can be optimized to fit the desired property of the mixture. When this optimization was done for the measurements listed in Table I the parameters $\xi = 0.710$ and $\eta = 0.999$ were obtained. The curves shown in Fig. 1 were calculated using these parameters. The above method provides a way of testing the internal consistency of the set of H_m^E measurements. [If measurements at one of the experimental temperatures are wrong, it follows that none of the calculated curves will be a good fit to experiment.] However, Fig. 1 shows that a good fit to all the measurements is obtained, and the fit to the measurements at 650.7 K is particularly pleasing. As all the measurements are fitted to within experimental error, we are reassured that our measurements at near-critical pressures are consistent with the rest of the data set.

$\begin{array}{c} H_{\rm m}^{\rm E} \\ ({\rm J} \cdot {\rm mol}^{-1}) \end{array}$	1847 2349	2909				10047	10164			4174	4581			2957
p (MPa)	7.95 11.05	11.90				24.80	25.30			24.40	25.30			24.95
$H_{\mathrm{m}}^{\mathrm{E}}(\mathrm{J}\cdot\mathrm{mol}^{-1})$	1511 1728	1737	1844	2946		8468	10687	10375	9953	2651	2659	3330	3592	2950
p (MPa)	7.20 9.20	9.25	10.10	14.50		22.70	23.35	23.65	24.55	19.80	19.90	22.10	23.10	23.00
$\frac{H_{\rm m}^{\rm E}}{({\rm J}\cdot{\rm mol}^{-1})}$	1287 1311	1336	1575	1591	2085	3550	4003	4479	5690	2017	2095	2251	2478	1893
p (MPa)	6.30 7.70	8.15	8.60	10.35	12.35	19.15	20.00	21.26	21.60	17.25	17.85	18.30	18.80	19.35
$\frac{H_{\rm m}^{\rm E}}{({\rm J}\cdot{\rm mol}^{-1})}$	1133 1073	1199	1152	1119	1567	1839	2329	2775	2881	1730	1696	1893	2089	1752
p (MPa)	5.70 6.65	7.15	7.25	8.25	10.30	14.15	16.20	17.35	18.10	15.45	15.65	16.36	17.20	18.30
$\frac{H_{\rm m}^{\rm E}}{({\rm J}\cdot{\rm mol}^{-1})}$	1009 681	1028	897	610	847	1015	1385	1465	1813	836	1102	1245	1455	1374
p (MPa)	5.15 5.05	5.85	6.15	5.25	6.80	9.55	11.60	12.25	12.65	9.50	11.25	12.40	14.15	15.35
T(K)	573.7 598.0			622.9		650.7				674.2				699.4

Table I. The Excess Molar Enthalpy $H_{\rm m}^{\rm E}$ of $(0.5H_2\,{\rm O}+0.5\,{\rm C}_2\,{\rm H_6(g)})$

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Excess Enthalpy of (Steam + Ethane)

A revealing way to examine the measurements is to plot $(H_{\rm m}^{\rm E}/p)$ against the pressure p, and the previous $H_{\rm m}^{\rm E}$ measurements [4] were plotted in this way. On this type of plot, the intercepts at zero pressure are simply related to the excess isothermal Joule–Thomson coefficient of the gas mixture by the equation [19]

$$\lim(p \to 0)(H_{\rm m}^{\rm E}/p) = x(1-x)(2\phi_{12} - \phi_{11} - \phi_{22}) \tag{7}$$

where the isothermal Joule–Thomson coefficients ϕ are related to the second virial coefficients by the equation

$$\phi = (B - TdB/dT) \tag{8}$$

Figure 2 shows some of the measurements made previously [4] at pressures below 12 MPa and the measurements listed in Table I. The curves were calculated from the two-fluid corresponding-states model. The figure shows overlap between the earlier measurements and those listed in Table I. The two sets of measurements were obtained using calorimeters of different design, and the generally good consistency between the two sets of measurements is reassuring. The figure shows that the model is a poor fit



Fig. 2. The quantity $(H_m^{\rm m}/p)$ calculated from values of $H_m^{\rm m}$ (0.5H₂O + 0.5C₂H₆)(g), plotted against the pressure *p*. (\bigcirc) This work (Table I); (\triangle) Ref. 4; (—) calculated from the two-fluid corresponding-states model.

to the measurements at pressures below 6 MPa. Both the virial equation of state [20] and the cubic equation of state [5] reported previously fit these low-pressure measurements to almost within the experimental error and are better equations for low pressure calculations. At pressures above 6 MPa, the two-fluid corresponding-states model is a good fit to the measurements. The figure reveals that at 650.7 and 699.4 K some of the points in the range 14 to 19 MPa are low, but the measurements in the range 6 to 12 MPa reported earlier are well fitted by the corresponding-states model.

4. COMPOSITION DEPENDENCE OF H_{m}^{E}

The composition dependence of $H_{\rm m}^{\rm E}$ is of interest, but we made no measurements other than those at x = 0.5. Experience with other near-critical mixtures [21] leads us to expect that in the gas phase at low densities,



Fig. 3. Values of the excess enthalpy H_m^E for $(xH_2O + (1-x)C_2H_6)(g)$, calculated over a range of pressure and composition from the two-fluid corresponding-states model.

the $H_{\rm m}^{\rm E}(x)$ curves will not be far from parabolic, but as the pressure increases the parabola will become progressively more skewed. To investigate this, we used the two-fluid corresponding states model to calculate the composition dependence of $H_{\rm m}^{\rm E}$ at selected pressures, concentrating particularly on the region between 20 and 25 MPa, where the $H_{\rm m}^{\rm E}$ curve at 650.7 K rises rapidly. A selection of $H_{\rm m}^{\rm E}(x)$ curves at this temperature is shown in Fig. 3. We see that at 10 MPa the $H_{\rm m}^{\rm E}(x)$ curve is a slightly skewed parabola, with a maximum at x = 0.65, and at 25 MPa the curve is extremely skewed, with a maximum at x = 0.82. At this pressure, the calculated value of $H_{\rm m}^{\rm E}$ at x = 0.82 is approximately 30% larger than that at x = 0.5. Although the curves shown in Fig. 3 are correct at x = 0.5, they may be inaccurate at high values of x, as the fitting parameters are slightly composition dependent. The curves are a good guide as to what shape the $H_{\rm m}^{\rm E}(x, p, T)$ surface might have, though they may well be in error by $\pm 5\%$ or more at the water-rich end.

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